Mathematica 11.3 Integration Test Results

Test results for the 19 problems in "4.4.1.3 (d cos)^m (a+b cot)^n.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]}{i + \text{Cot}[x]} \, dx$$

Optimal (type 3, 18 leaves, 8 steps):

$$-i \,\text{ArcTanh}[\text{Sin}[x]] - \text{Cos}[x] + i \,\text{Sin}[x]$$

Result (type 3, 44 leaves):

$$-\text{Cos}[x] + i \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x]\right)$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]^3}{i + \text{Cot}[x]} \, dx$$

Optimal (type 3, 22 leaves, 8 steps):

$$\frac{1}{2} i \,\text{ArcTanh}[\text{Sin}[x]] + \text{Sec}[x] - \frac{1}{2} i \,\text{Sec}[x] \,\text{Tan}[x]$$

Result (type 3, 48 leaves):

$$-\frac{1}{2} i \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sec}[x] \left(2 i + \text{Tan}[x]\right)\right)$$

Problem 11: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cos}[x]^4}{a + b \,\text{Cot}[x]} \, dx$$

Optimal (type 3, 126 leaves, 8 steps):

$$\frac{a \left(3 a^4 - 6 a^2 b^2 - b^4\right) x}{8 \left(a^2 + b^2\right)^3} - \frac{a^4 b \,\text{Log}[b \,\text{Cos}[x] + a \,\text{Sin}[x]]}{\left(a^2 + b^2\right)^3} + \frac{\left(4 b \left(2 a^2 + b^2\right) + a \left(5 a^2 + b^2\right) \text{Cot}[x]\right) \text{Sin}[x]^2}{8 \left(a^2 + b^2\right)^2} - \frac{\left(b + a \,\text{Cot}[x]\right) \text{Sin}[x]^4}{4 \left(a^2 + b^2\right)}$$

Result (type 3, 179 leaves):

$$\frac{1}{32\left(a^2+b^2\right)^3}\Big(12 a^5 x-32 i a^4 b x-24 a^3 b^2 x-4 a b^4 x+32 i a^4 b \operatorname{ArcTan}[\operatorname{Tan}[x]]-4 b\left(3 a^4+4 a^2 b^2+b^4\right) \operatorname{Cos}[2 x]- a^4 b \operatorname{Cos}[4 x]-2 a^2 b^3 \operatorname{Cos}[4 x]-b^5 \operatorname{Cos}[4 x]-16 a^4 b \operatorname{Log}\left[(b \operatorname{Cos}[x]+a \operatorname{Sin}[x])^2\right]+ 8 a^5 \operatorname{Sin}[2 x]+8 a^3 b^2 \operatorname{Sin}[2 x]+a^5 \operatorname{Sin}[4 x]+2 a^3 b^2 \operatorname{Sin}[4 x]+a b^4 \operatorname{Sin}[4 x]\Big)$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cos}[x]^2}{a+b \operatorname{Cot}[x]} \, dx$$

Optimal (type 3, 73 leaves, 7 steps):

$$\frac{a\left(a^2-b^2\right) x}{2\left(a^2+b^2\right)^2}-\frac{a^2 b \operatorname{Log}[b \operatorname{Cos}[x]+a \operatorname{Sin}[x]]}{\left(a^2+b^2\right)^2}+\frac{(b+a \operatorname{Cot}[x]) \operatorname{Sin}[x]^2}{2\left(a^2+b^2\right)}$$

Result (type 3, 82 leaves):

$$\frac{1}{4\left(a^2+b^2\right)^2}\Big(4 i a^2 b \operatorname{ArcTan}[\operatorname{Tan}[x]]-b\left(a^2+b^2\right) \operatorname{Cos}[2 x]+ a\left(2(a-i b)^2 x-2 a b \operatorname{Log}\left[(b \operatorname{Cos}[x]+a \operatorname{Sin}[x])^2\right]+\left(a^2+b^2\right) \operatorname{Sin}[2 x]\right)\Big)$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[x]^3}{a+b \operatorname{Cot}[x]} \, dx$$

Optimal (type 3, 79 leaves, 9 steps):

$$\frac{\operatorname{ArcTanh}[\operatorname{Sin}[x]]}{2 a}+\frac{b^2 \operatorname{ArcTanh}[\operatorname{Sin}[x]]}{a^3}+ \frac{b \sqrt{a^2+b^2} \operatorname{ArcTanh}\left[\frac{a \operatorname{Cos}[x]-b \operatorname{Sin}[x]}{\sqrt{a^2+b^2}}\right]}{a^3}-\frac{b \operatorname{Sec}[x]}{a^2}+\frac{\operatorname{Sec}[x] \operatorname{Tan}[x]}{2 a}$$

Result (type 3, 192 leaves):

$$-\frac{1}{4 a^3}\Bigg(8 b \sqrt{a^2+b^2} \operatorname{ArcTanh}\left[\frac{-a+b \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]+ \operatorname{Sec}[x]^2\Big(4 a b \operatorname{Cos}[x]+a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]-\operatorname{Sin}\left[\frac{x}{2}\right]\right]+2 b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]-\operatorname{Sin}\left[\frac{x}{2}\right]\right]+ \left(a^2+2 b^2\right) \operatorname{Cos}[2 x]\left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]-\operatorname{Sin}\left[\frac{x}{2}\right]\right]-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]+\operatorname{Sin}\left[\frac{x}{2}\right]\right]\right)- a^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]+\operatorname{Sin}\left[\frac{x}{2}\right]\right]-2 b^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]+\operatorname{Sin}\left[\frac{x}{2}\right]\right]-2 a^2 \operatorname{Sin}[x]\Big)\Bigg)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[x]}{1+2\operatorname{Cot}[x]}\,dx$$

Optimal (type 3, 25 leaves, 6 steps):

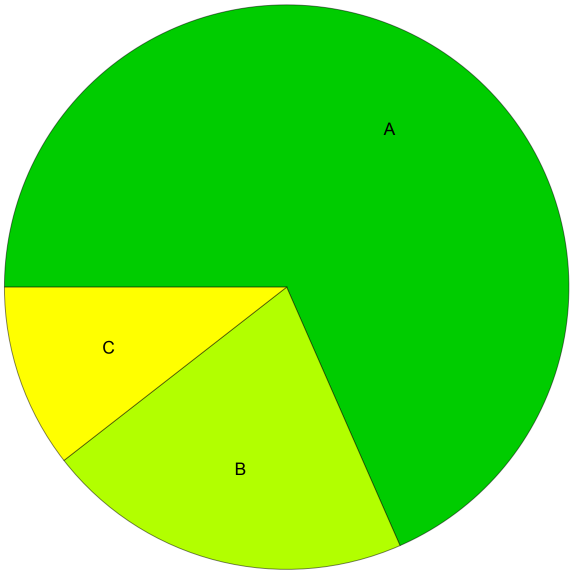
$$\frac{2\operatorname{ArcTanh}\left[\frac{\operatorname{Cos}[x]-2\operatorname{Sin}[x]}{\sqrt{5}}\right]}{\sqrt{5}}+\operatorname{ArcTanh}[\operatorname{Sin}[x]]$$

Result (type 3, 57 leaves):

$$\frac{4\operatorname{ArcTanh}\left[\frac{1-2\operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{5}}\right]}{\sqrt{5}}-\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]-\operatorname{Sin}\left[\frac{x}{2}\right]\right]+\operatorname{Log}\left[\operatorname{Cos}\left[\frac{x}{2}\right]+\operatorname{Sin}\left[\frac{x}{2}\right]\right]$$

Summary of Integration Test Results

19 integration problems



A - 13 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts